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On rainbow antimagic coloring and local edge antimagic coloring of graphs

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Abstract

Let G be a connected graph of order n. Let $f : V(G) \to \{1, 2, \ldots, n\}$ be a bijection and for every $uv \in E(G)$ consider $w_f(uv) = f(u) + f(v)$ as a coloring of the edge. For a pair of vertices u and v, they are connected by a rainbow path if there exists a path from u to v such that the edges have pairwise distinct colors from w_f. The bijection $f : V(G) \to \{1, 2, \ldots, n\}$ is a *rainbow antimagic coloring* if for every two vertices there exists a rainbow path. Meanwhile, that bijection $f : V(G) \to \{1, 2, \ldots, n\}$ is *local edge antimagic coloring* if every two adjacent edges have distinct weights. The *rainbow antimagic connection number* $rac(G)$ and *local edge antimagic chromatic number* $\chi'_{lea}(G)$ is the minimum number of distinct edge weights over all rainbow antimagic coloring and local edge antimagic coloring, respectively.

We investigate the relationship between $rac(G)$ and $\chi'_{lea}(G)$. We prove $\chi'_{lea}(G) \leq rac(G)$ for all graphs and provide conditions for equality. Graphs with diameter at most 2 or satisfying $rac(G) = \Delta(G)$ achieve equality. We construct a family A_d with arbitrarily large diameter d where $\Delta(A_d) = \chi'_{lea}(A_d) = rac(A_d)$, and a family $H_n = K_{n,n} - nK_2$ of diameter 3 where $\chi'_{lea}(H_n) = rac(H_n) = 2n - 3 > \Delta(H_n)$. These results present hints to the full characterization of graphs G with $\chi'_{lea}(G) = rac(G)$.

Keywords: rainbow antimagic coloring, rainbow antimagic connection number, local edge antimagic coloring, local edge antimagic chromatic number
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1. Introduction

The rainbow connection number of a graph, denoted by $rc(G)$, was introduced by Chartrand *et al.* [10]. Rainbow connection number of a connected graph G is the minimum number of colors needed to edge-color G such that every pair of vertices is joined by a rainbow path, that is a path whose edges receive pairwise distinct colors. The study of rainbow connection originally motivated by secure information transfer across networks, and thus the parameter has since attracted considerable attention in structural graph theory and combinatorial optimization [10, 22, 7]. Trivially bounded below by the diameter ($diam(G) \leq rc(G)$), exact values and sharp upper bounds have been established for numerous graph classes. In particular, trees satisfy $rc(T) = n - 1$ [10], cycles satisfy $rc(C_n) = \lceil n/2 \rceil$ for $n \geq 4$ [10], while complete graphs with $n \geq 3$ satisfy $rc(K_n) = 1$ [10]. Chandran *et al.* [8] proved that for any bridgeless graph, $rc(G) \leq \text{rad}(G)(\text{rad}(G) + 2)$, where $\text{rad}(G)$ denotes the radius. Moreover, Krivelevich and Yuster [19] demonstrated that for any connected graph on n vertices with minimum degree δ, a probabilistic argument yields $rc(G) \leq \frac{3n}{\delta+1} + 3$. Caro *et al.* [6] further refined these bounds by relating the rainbow connection number to various parameters including the size of a minimum edge cover. In addition, Schiermeyer [26] established upper bounds in terms of the number of cut-edges. The computational complexity of rainbow connection was settled by Chakraborty *et al.* [7], who proved that deciding whether $rc(G) \leq 2$ is NP-complete even for graphs of diameter 2, while hardness of approximation was further studied by Ananth and Nasre [4]. Extensions to vertex coloring were initiated by Krivelevich and Yuster [19]. Ekstein *et al.* [13] investigated the rainbow connection number of special graph classes including planar graphs and graphs with given treewidth. A survey of numerous results on rainbow connection, rainbow k-connectivity, and algorithmic aspects is given by Li and Sun [21].

The rainbow antimagic connection number, denoted by $rac(G)$, was introduced by Septory *et al.* [27] as a natural combination of the rainbow connection number and the antimagic labeling of graphs. Given a connected graph G of order n, a bijective function $f : V(G) \to \{1, 2, \ldots, n\}$ is called a *rainbow antimagic coloring* of G if every pair of distinct vertices $u, v \in V(G)$ is connected by a rainbow path, where the edge weights are defined by $w_f(uv) = f(u) + f(v)$ for each edge $uv \in E(G)$. The *rainbow antimagic connection number* $rac(G)$ is the minimum number of colors taken over all rainbow antimagic colorings of G [27]. Septory et al. [27] conduct the study to consider several special graph families, determining the rainbow antimagic connection number for lemon graphs, firecracker graphs, complete bipartite graphs, double stars, while also establishing general bounds including $diam(G) \leq rac(G)$. Sulistyo et al. [29] extended the study to several special graph families, determining the rainbow antimagic connection number for paths, ladders, triangular ladders and triangular book graphs. For trees, Dafik et al. [11] established that $rac(T) = n - 1$ for any tree T of order $n \geq 2$, providing one of the first exact values for this parameter. They also determined the rainbow antimagic connection number of complete graphs, cycles, wheel graphs, fan graphs and friendship graphs. Furthermore, they characterized graphs G with $rac(G) = 1$ and $rac(G) = 2$ [11]. Moreover, several graphs such as lollipop graphs, book graphs, dutch windmills, flower pot graphs and dragonfly graphs are considered by Budi et al. [5]. Maghfiroh et al. [23] studied the rainbow antimagic connection number of double wheel graphs $2C_n + K_1$ and parachute graphs, providing exact formulas for these families. The parameter was further investigated for corona products of graphs by Septory et al. [28], where exact values were obtained for the corona product involving stars and other graph such as paths, stars and friendship graphs. Dafik et al. [12] contributed results on octopus graphs, sandat graphs, sunflower graphs and semi-Jahangir graphs. Adawiyah *et al.* [1] determined snail graphs S_n, coconut root graphs $Cr_{n,m}$, fan stalk graphs Kt_n, and lotus graphs Lo_n, determining their respective rainbow antimagic connection numbers.

On the other hand, the concept of local edge antimagic coloring was introduced by Agustin *et al.* [2] as a relaxation of the classical edge antimagic labeling. In an edge antimagic labeling, first studied by Hartsfield and Ringel [18], a bijection $f : V(G) \to \{1, 2, \dots, |V(G)|\}$ is assigned to the vertices such that the induced edge weights $w(uv) = f(u)+f(v)$ are pairwise distinct for all edges in the graph. Relaxing this global distinctness condition, Agustin et al. [2] required only that adjacent edges receive distinct weights, formally defining a bijection $f : V(G) \to \{1, 2, \dots, |V(G)|\}$ to be a *local edge antimagic coloring* if for every pair of adjacent edges $uv, vx \in E(G)$, the induced weights satisfy $w(uv) \neq w(vx)$. The *local edge antimagic chromatic number*, denoted by $\chi'_{lea}(G)$, is the minimum number of distinct edge weights taken over all such labelings of G [2]. Agustin et al. [2] established foundational results, showing that $\chi'_{lea}(P_n) = 2$ for paths of order $n \geq 3$, $\chi'_{lea}(C_n) = 3$ for cycles of order $n \geq 5$, and $\chi'_{lea}(K_{1,n}) = n$ for star graphs. Rajkumar and Nalliah [25] investigated $\chi'_{lea}(G)$ for friendship graphs, wheel graphs, fan graphs, helm graphs, and flower graphs. Hadiputra and Maryati [17] contributed results on the characterization of connected graphs G with $\chi'_{lea}(G) = 1$ and $\chi'_{lea}(G) = 2$. They also studied the parameter for copies of paths and comb products involving stars, establishing exact formulas for these families [17]. The study was extended, where Chandra and Silaban [9] determined χ'_{lea} for comb products of various graphs involving paths. For some chains of paths and cycles, the local edge antimagic chromatic number is determined by Walfried *et al.* [30]. Further investigations on the local edge antimagic chromatic number of join product of graphs can be seen in [24]. There are many variations of antimagic graphs, for example see [15, 16, 20, 3]. For more information about graph labeling, refer to [14].

Both rainbow antimagic coloring and local edge antimagic coloring use the same kind of labeling bijection and weight function, and this provide the motivation to investigate the relationship between two parameters: the rainbow antimagic connection number $rac(G)$ and the local edge antimagic chromatic number $\chi'_{lea}(G)$. After establishing foundational results, including a proof that every graph admits a rainbow antimagic coloring and the general inequality $\chi'_{lea}(G) \leq rac(G)$, we turn to the central problem of characterizing graphs for which these two parameters coincide. We show that graphs with diameter at most 2 always satisfy $\chi'_{lea}(G) = rac(G)$, and that the same equality holds whenever $rac(G) = \Delta(G)$. Our main contributions address two natural follow-up questions. First, we prove that the equality does not require bounded diameter by constructing, for any integer $d \geq 5$, a graph A_d with arbitrarily large diameter d such that $\Delta(A_d) = \chi'_{lea}(A_d) = rac(A_d)$. Second, we demonstrate that the equality can occur even when both parameters strictly exceed the maximum degree and the diameter exceeds 2, by analyzing the family $H_n = K_{n,n} - nK_2$, which has diameter 3 and satisfies $\chi'_{lea}(H_n) = rac(H_n) = 2n - 3 > n = \Delta(H_n)$.

2. Foundational Properties

Throughout this paper, we use the notation $[a, b] = \{k \mid k \in \mathbb{N}, a \leq k \leq b\}$ for any $a, b \in \mathbb{N}$. In [11], the author determined the rainbow antimagic connection number for trees.

Theorem 2.1. *[11] For any tree T of order $n \geq 2$, we have $rac(T) = n - 1$.*

In their work, it is shown that rainbow antimagic coloring exists for graphs with certain conditions. However, this restrictions are not necessary since we can show the same result without them.

Theorem 2.2. *All graphs admit rainbow antimagic. In particular, $rac(G) \leq |E(G)|$.*

Proof. Let G be any graph of order n and let T be a spanning tree of G. Let $f : V(T) \to [1, n]$ be a rainbow antimagic coloring of T given by Theorem 2.1. Now, consider f as a mapping from $V(G)$ to $[1, n]$ and we will show that f is a rainbow antimagic coloring of G.

For every two distinct vertices $u, v \in V(G)$, there exists a certain path P from u to v in the spanning tree T. Since f is a rainbow antimagic coloring of T, it holds that P is a rainbow path. This shows that every two vertices in G is rainbow-connected. Hence, f is a rainbow antimagic coloring of G. Observe that f uses at most $|E(G)|$ colors, which implies $rac(G) \leq |E(G)|$. □

Since we have shown that all graphs admit rainbow antimagic coloring, then $rac(G)$ is well-defined for any graph G. Therefore, we can freely write for every graph G satisfies the following inequality:

$$diam(G) \leq rc(G) \leq rac(G) \leq |E(G)|. \tag{1}$$

Now, we can show the following bound between local edge antimagic chromatic number and rainbow antimagic connection number of a graph.

Theorem 2.3. *Let G be a graph of order $n \geq 2$. Let $f : V(G) \to [1, n]$ be a bijection and let $w_f : E(G) \to \mathbb{N}$ be a map defined by $w_f(uv) = f(u) + f(v)$. Then for every two adjacent edges $uv, vx \in E(G)$, we have $w_f(uv) \neq w_f(vx)$. Furthermore, $\chi'_{lea}(G) \leq rac(G)$.*

Proof. Given that f is a bijection, it follows that

$$f(u) \neq f(x) \iff f(u) + f(v) \neq f(v) + f(x) \iff w_f(uv) \neq w_f(vx).$$

Now, let $f : V(G) \to [1, n]$ be a rainbow antimagic coloring of G which induces k colors. By above discussion, we obtain f is a local edge antimagic coloring of G which induces k colors as well. In particular, if we choose $k = rac(G)$, we obtain $\chi'_{lea}(G) \leq rac(G)$. □

Let $\Delta(G)$ be the largest vertex degree of a graph G. As a consequence of Theorem 2.3, we also have the following inequality:

$$\Delta(G) \leq \chi'(G) \leq \chi'_{lea}(G) \leq rac(G). \tag{2}$$

Table 1: Graphs G with known $rac(G)$ and $\chi'_{lea}(G)$ to be distinct.

Graphs	$rac(G)$	$\chi'_{lea}(G)$	References
Paths $P_n, n \geq 3$	$n-1$	2	[29], [2]
Cycles $C_n, n \geq 5, n \equiv 1,2 \pmod 4$	$\lceil \frac{n}{2} \rceil$	3	[11], [2]
Cycles $C_n, n \geq 5, n \equiv 0,3 \pmod 4$	$\lceil \frac{n}{2} \rceil$ or $\lceil \frac{n}{2} \rceil + 1$	3	[11], [2]
Ladders $P_2 \square P_n$, n is odd	n	3	[29], [2]
Ladders $P_2 \square P_n$, n is even	n or $n+1$	3	[29], [2]
Broom graphs $Amal(P_n, S_m; v_1, c)$,	$m+n$	$m+1$	[11], [17, Corollary 4]
Lollipop graphs $Amal(P_n, C_3; v_1, v_1)$,	$n+2$	3	[5], [17, Corollary 4]
Regular caterpillar $P_n \triangleright S_m$	$n(m+1)-1$	$\leq m+n-1$	[11], [17, Corollary 5]

3. Comparison Analysis

It is known that the difference of $rac(G) - \chi'_{lea}(G)$ can be arbitrary large for several family of graphs. In Table 1, we provide some graphs whose both rainbow antimagic connection number and local edge antimagic chromatic number are known to be distinct.

Now, it would be interesting to see which graphs G satisfying $\chi'_{lea}(G) = rac(G)$. One possible condition is for the graph to have small diameter.

Lemma 3.1. *If $diam(G) \leq 2$, then $\chi'_{lea}(G) = rac(G)$.*

Proof. Since $\chi'_{lea}(G) \leq rac(G)$ due to Theorem 2.3, it is sufficient to show $rac(G) \leq \chi'_{lea}(G)$. Let n be the order G and let $f : V(G) \to [1, n]$ be a local edge antimagic coloring of G which induces k colors. Let $w_f : E(G) \to \mathbb{N}$ be the induced weights of f. By Theorem 2.3, for every two adjacent edges $uv, vx \in E(G)$, $w_f(uv) \neq w_f(vx)$. This implies every path of length two in G is a rainbow path. Since $diam(G) \leq 2$, therefore every two distinct vertices of G has a rainbow path. This shows that f is a rainbow antimagic coloring of G with k colors as well. If we choose $k = \chi'_{lea}(G)$, it follows that $rac(G) \leq \chi'_{lea}(G)$. □

Table 2: Graphs G with known $rac(G) = \chi'_{lea}(G)$ where $diam(G) \leq 2$.

Graphs	$rac(G) = \chi'_{lea}(G)$	$\Delta(G)$	References
Complete graphs $K_n, n \geq 3$	$2n-3$	$n-1$	[11], [2]
Wheel graphs $C_n + K_1, n \geq 5$	n	n	[11], [25]
Double wheel graphs $2C_n + K_1, n \geq 3$	$2n$	$2n$	[23]
Fan graphs $P_n + K_1, n \geq 3$	n	n	[11], [25]
Friendship graphs $nK_2 + K_1, n \geq 2$	$2n$	$2n$	[29], [2]
Triangular book graphs $K_1 + K_{1,m}$	$m+2$	$m+1$	[28]
Complete bipartite graphs $K_{m,n}$	$m+n-1$	$\max\{m, n\}$	[24]
Join product of graphs $G + H$	varies	varies	[24]

Based on this result, we obtain several families of graphs G satisfying $\chi'_{lea}(G) = rac(G)$. We list them in Table 2. Among graphs G with $rac(G) = \chi'_{lea}(G)$, although previous results present a lot of family of graphs with diameter at most 2, there exists other graphs with diameter 3 or 4. Another sufficient condition to satisfy the equality is ensuring the rainbow antimagic connection number of the graph equals the largest degree of the graph.

Corollary 3.1. *If $rac(G) = \Delta$, then $\chi'_{lea}(G) = rac(G)$.*

Proof. Follows from Equation (2). □

Therefore, we have another table for graphs satisfying the equality, presented in Table 3.

Table 3: Graphs G with known $rac(G) = \chi'_{lea}(G)$ where $rac(G) = \Delta(G)$.

Graphs	$rac(G) = \chi'_{lea}(G)$	Diameter	References
Book graphs $K_2 \square K_{1,m}$, m is even	$m+1$	3	[5]
Flower pot graphs $C_3 S_n$, $n \geq 2$	$n+1$	3	[5]
Dutch windmill $D_n^{(3)}$, $n \geq 3$	$2n$	4	[5]
Semi-Jahangir SJ_n, $n \geq 3$	n	4	[12]
Lotus graphs Lo_n, $n \geq 2$	$n+1$	4	[1]

However, among results that we found, these are only graphs with diameter at most 4. Therefore, an interesting question is whether graphs G satisfying $rac(G) = \chi'_{lea}(G)$ must have bounded diameter. In the proceeding theorem, we prove that this is not the case.

Theorem 3.1. *There exists a graph G where $\Delta(G) = \chi'_{lea}(G) = rac(G)$ and $diam(G) = d$ for any $d \geq 2$.*

Proof. For $d \in \{2, 3, 4\}$, refer to Table 2 and 3. Now, it is sufficient to only consider $d \geq 5$. Fix an integer $d \geq 5$ and define a graph A_d with the vertex set and the edge set

$$V(A_d) = \{y_i \mid i \in [1, 3d-5]\} \cup \{x_i \mid i \in [1, 2d-1]\},$$
$$E(A_d) = \{x_1 y_i \mid i \in [1, 3d-5]\} \cup \{x_i x_{i+1} \mid i \in [1, 2d-2]\} \cup \{x_1 x_{2d-1}\}.$$

It is an exercise to check whether $diam(A_d) = d$, $\Delta(A_d) = 3d-3$ and $|V(A_d)| = 5d-6$. Now, consider a map $f : V(A_d) \to [1, 5d-6]$ given by

$$f(x_i) = \begin{cases} 2d-2, & \text{for } i = 1, \\ \frac{i}{2}, & \text{for } i \in [2, 2d-2], i \text{ is even}, \\ 4d-6+\frac{i+1}{2}, & \text{for } i \in [3, 2d-1], i \text{ is odd}, \end{cases}$$

$$f(y_i) = \begin{cases} d+i-1, & \text{for } i \in [1, d-2], \\ d+i, & \text{for } i \in [d-1, 3d-5]. \end{cases}$$

It is a routine to check that f is a bijection. This map f induces a weight function $w_f : E(A_d) \to \mathbb{N}$ where

$$w_f(x_1y_i) = \begin{cases} 3d+i-3, & \text{for } i \in [1, d-2], \\ 3d+i-2, & \text{for } i \in [d-1, 3d-5], \end{cases}$$
$$w_f(x_ix_{i+1}) = \begin{cases} 2d-2+\frac{i}{2}, & \text{for } i = 1, \\ 4d-5+i, & \text{for } i \in [2, 2d-2], \end{cases}$$
$$w_f(x_1x_{2d-1}) = 7d-8.$$

Since $w_f(x_ix_{i+1}) = w_f(x_1y_{i+d-3})$ for $i \in [2, 2d-2]$, the map w_f induces only $3d-3 = \Delta(A_d)$ colors. To show that every two distinct vertices have a rainbow path, observe that:

- Every two distinct edges x_ix_{i+1} and $x_{i'}x_{i'+1}$ (or one of them is x_1x_{2d-1}) has distinct colors. Hence, every two distinct vertices x_j and $x_{j'}$ has a rainbow path.
- Every two distinct edges x_1y_i and $x_1y_{i'}$ has distinct colors. Hence, every two distinct vertices y_j and $y_{j'}$ has a rainbow path.
- Consider x_i and y_j where $i \neq 1$. There are always two distinct paths that starts from x_i and ends at y_j. Since the color x_1y_j only appeared once among the edges x_kx_{k+1}, there exists a rainbow path from x_i to y_j.

Therefore, f is a rainbow antimagic coloring which implies $rac(A_d) \leq 3d-3 = \Delta(A_d)$. Since $\Delta(A_d) \leq rac(A_d)$ from Equation (2), we obtain $rac(A_d) = \Delta(A_d)$. By Corollary 3.1, we also obtain $rac(A_d) = \chi'_{lea}(A_d)$. □

We present an example of A_5 along with its rainbow antimagic coloring in Figure 1.

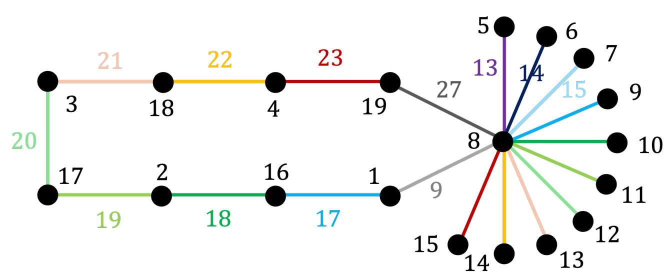

Figure 1: The graph A_5 with $\Delta(A_5) = \chi'_{lea}(A_5) = rac(A_5) = 12$.

Next, the follow-up question is if we have $\chi'_{lea}(G) > \Delta(G)$, is it still possible to have $rac(G) = \chi'_{lea}(G)$? From Table 2, we see complete graphs and complete bipartite graphs as answers to this question but only for diameter at most 2. Hence, in the following theorem, we show an existence of a graph G whose diameter is 3 with $rac(G) = \chi'_{lea}(G)$.

Theorem 3.2. *Let $n \geq 3$ be positive integer and let $H_n = K_{n,n} - nK_2$. Then $\chi'_{lea}(H_n) = rac(H_n) = 2n - 3$.*

Proof. Let H_n be a graph with the vertex set and edge set

$$V(H_n) = \{x_i, y_i \mid i \in [1, n]\},$$
$$E(H_n) = \{x_i y_j \mid i, j \in [1, n], i \neq j\}.$$

We define a map $f : V(H_n) \to [1, 2n]$ where

$$f(x_i) = i, \qquad \text{for } i \in [1, n],$$
$$f(y_i) = i + n, \qquad \text{for } i \in [1, n].$$

Observe that f is a bijective map. Then, we have induced weight function given by

$$w_f(x_i y_j) = i + j + n, \qquad \text{for } i \in [1, n], j \in [1, n], i \neq j.$$

To see why this is a rainbow antimagic coloring, we only need to check two vertices of distance 3 which is the pair x_i and y_i for every $i \in [1, n]$. Consider the path $x_i y_j x_k y_i$ where $i, j, k \in [1, n]$ are pairwise distinct. Therefore, the colors $i + j + n, j + k + n, k + i + n$ are also pairwise distinct. This shows that every two vertices of H_n has a rainbow path. Henceforth, $rac(H_n) \leq 2n - 3$.

Now, we want to show that $\chi'_{lea}(H_n) \geq 2n - 3$. Let f be any local edge antimagic coloring of H_n. Without loss of generality, let $\min_{i\in[1,n]}\{f(x_i)\} < \min_{i\in[1,n]}\{f(y_i)\}$. Let $a \in [1, n]$ such that $f(x_a) < f(x_i)$ for every $i \in [1, n], i \neq a$. Analogously, let $b \in [1, n]$ such that $f(y_b) > f(y_i)$ for every $i \in [1, n], i \neq b$. Define

$$S = \{f(x_a) + f(y_i) \mid i \in [1, n], i \neq a\},$$
$$T = \{f(x_i) + f(y_b) \mid i \in [1, n], i \neq b\}.$$

Observe that each S and T contains $n - 1$ strictly increasing numbers. Furthermore, $\max S = f(x_a) + f(y_b) = \min T$ which implies $S \cap T = \{f(x_a) + f(y_b)\}$. Therefore, in any local edge antimagic coloring of H_n, there exists at least $2n - 3$ colors. In other words, $\chi'_{lea}(H_n) \geq 2n - 3$. By Theorem 2.3, it follows that $rac(H_n) = \chi'_{lea}(H_n) = 2n - 3$. □

An illustration of rainbow antimagic coloring of H_4 is given in Figure 2.

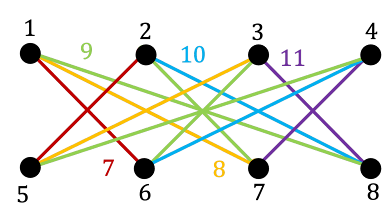

Figure 2: The graph H_4 with $\chi'_{lea}(H_4) = rac(H_4) = 5 > \Delta(H_4)$.

4. Concluding Remarks

In this paper, we have established fundamental connections between the rainbow antimagic connection number $rac(G)$ and the local edge antimagic chromatic number $\chi'_{lea}(G)$ for arbitrary graphs. We proved that every graph admits a rainbow antimagic coloring, thereby ensuring that $rac(G)$ is well-defined for all graphs, and established the general inequality $\chi'_{lea}(G) \leq rac(G)$.

Our aim is focused on characterizing graphs for which these two parameters coincide. We identified that graphs with diameter at most 2 always satisfy $\chi'_{lea}(G) = rac(G)$, which includes several well-known families such as complete graphs, wheels, fans, friendship graphs, and complete bipartite graphs. Furthermore, we showed that the condition $rac(G) = \Delta(G)$ also guarantees equality of the two parameters, leading to additional examples with diameters 3 and 4, including book graphs, flower pot graphs, Dutch windmills, semi-Jahangir graphs, and lotus graphs.

A natural question arising from these observations is whether graphs satisfying $\chi'_{lea}(G) = rac(G)$ must have bounded diameter. We answered this in the negative by constructing, for any integer $d \geq 5$, there exists a graph A_d with $\Delta(A_d) = \chi'_{lea}(A_d) = rac(A_d)$ and $diam(A_d) = d$. This demonstrates that the equality can hold for graphs with arbitrarily large diameter when the rainbow antimagic connection number attains the trivial lower bound $\Delta(G)$.

We further explored whether the equality $\chi'_{lea}(G) = rac(G)$ can occur when $\chi'_{lea}(G) > \Delta(G)$ beyond graphs of diameter at most 2. By analyzing the graph $H_n = K_{n,n} - nK_2$, we exhibited a family of graphs with diameter 3 and $\chi'_{lea}(H_n) = rac(H_n) = 2n - 3 > \Delta(H_n) = n$, thus providing an affirmative answer to this question.

Based on our findings, we propose the following open problems for future research:

1. Characterize all graphs G for which $\chi'_{lea}(G) = rac(G)$.
2. Determine whether there exist graphs G with $rac(G) = \chi'_{lea}(G) > \Delta(G)$ and diameter $d \geq 4$.
3. Investigate structural properties that force or preclude the equality $\chi'_{lea}(G) = rac(G)$.

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