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New families of star-supermagic graphs

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Abstract

A simple graph G admits a $K_{1,n}$-*covering* if every edge in $E(G)$ belongs to a subgraph of G isomorphic to $K_{1,n}$. The graph G is $K_{1,n}$-supermagic if there exists a bijection $f : V(G) \cup E(G) \to \{1, 2, 3, \cdots, |V(G) \cup E(G)|\}$ such that for every subgraph H' of G isomorphic to $K_{1,n}$, $\sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e)$ is a constant and $f(V(G)) = \{1, 2, 3, \cdots, |V(G)|\}$. In such a case, f is called a $K_{1,n}$-supermagic labeling of G. In this paper, we give a method how to construct $K_{1,n}$-supermagic graphs from the old ones.

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1. Introduction

In this paper, we consider finite and simple graphs G with the vertex and edge sets $V(G)$ and $E(G)$, respectively. The number of vertices (edges) in the graph G is called *order* (*size*) of G. Let H be a given graph. An *edge-covering* of G is a family of subgraphs $H_1, \ldots, H_k$ such that each edge in $E(G)$ belongs to at least one of the subgraphs H_i, $1 \leq i \leq k$. Then it is said that G admits an $(H_1, \ldots, H_k)$-*(edge)covering*. If every H_i, $1 \leq i \leq k$, is isomorphic to the graph H, then G admits an H-covering. Suppose G admits an H-covering. A total labeling $f : V(G) \cup E(G) \to \{1, 2, 3, \ldots, |V(G) \cup E(G)|\}$ is called an H-*magic labeling* of G if for every subgraph H' of G isomorphic to H, $\sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e) = c_f$ is a constant. The constant c_f is called magic constant of the labeling f. An H-magic labeling f is called an

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H-*supermagic labeling* if $f(V(G)) = \{1, 2, 3, \ldots, |V(G)|\}$. A graph that admits H-(super)magic labelings is called H*-(super)magic*. In this paper, we consider such a labeling when H is a star $K_{1,n}$.

The H-(super)magic labeling was first introduced and studied by Gutiérrez and Lladó [3] in 2005 where H-supermagic labelings for stars, complete bipartite graphs, paths and cycles are considered. In [7], Lladó and Moragas studied C_n-supermagic labeling of some graphs. They proved that the wheel W_n, the windmill $W(r, k)$, and the prism $C_n \times P_2$ are C_h-supermagic for some h. Cycles-supermagic labeling of chain graphs kC_n-path, triangle ladders TL_n, grids $P_m \times P_n$, for $n = 2, 3, 4, 5$, fans F_n, and books B_n can be found in [8]. The complete results on these labelings can be found in [2].

For $H \cong P_2$, an H-supermagic graph is also called a *super edge-magic graph*. The notion of a super edge-magic graph was introduced by Enomoto at al [1] as a particular type of edge-magic graph given by Rosa [5]. For further information about (super) edge-magic graphs, see [2]. The H-magic labeling is related to a face-magic labeling of a plane graph introduced by Lih [6]. A total labeling f of a plane graph is said to be *face-magic* if for every positive integer s, all s-sided faces have the same weight. The weight of a face under the labeling f is the sum of labels carried by the edges and vertices surrounding it. Lih [6] allows different weights for different s. When a plane graph G contains only n-sided faces then face-magic labeling of G is also C_n-magic labeling. Other results about this labeling can be found in, for instance, [2].

In this paper, we give a method how to construct star-supermagic graphs from the old ones. Based on this, we have new families of star-supermagic graphs.

2. The Results

In this section, we propose a method for constructing new star-supermagic graphs from certain star-supermagic graphs. To do this, we need the the following notations. The sum of all vertex and edge labels on H (under a labeling f) is denoted by $\sum f(H)$. For any two integers $n < m$, the set of all consecutive integers from n to m is denoted by $[n, m]$. For any set $X \subset \mathbb{N}$, the set of natural numbers, we write $\Sigma X = \Sigma_{x \in X} x$. For any integer k, $X + k = \{x + k : x \in X\}$. Thus $k + [n, m]$ is the set of consecutive integers from $k + n$ to $k + m$. It is easy to check that $\Sigma(X + k) = k|X| + \Sigma X$. Furthermore, we also need the concept of a *k-balanced* set. $\mathbb{P} = \{X_1, X_2, \ldots, X_k\}$ is said to be an *equipartition* of a set of integers X if $X_1, X_2, \ldots, X_k$ are non-empty disjoint subsets of X whose union is X and, for $i \in [1, k]$, $|X_i| = \frac{|X|}{k}$. The set X is said to be *k-balanced* if there exists an equipartition $\mathbb{P} = \{X_1, X_2, \ldots, X_k\}$ of X with the property that $\Sigma X_i = \frac{\Sigma X}{k}$, $i \in [1, k]$.

Lemma 2.1. *For any positive integers k and m, the set $X = [1, 2km]$ is k-balanced.*

Proof. For every $i \in [1, k]$, define $A_i = [(i-1)m + 1, im]$ and $B_i = km + A_{k+1-i}$. For every $i \in [1, k]$, let $C_i = A_i \cup B_i$. It can be checked that for $i \neq j$, $C_i \cap C_j = \emptyset$, $\bigcup_{i=1}^{k} C_i = X$, and for $i \in [1, k]$, $|C_i| = 2m$. So, $\mathbb{P} = \{C_1, C_2, \ldots, C_k\}$ is an equipartition of X. Furthermore, for every $i \in [1, k]$, it can be checked that $\sum C_i = m(2km + 1)$. Thus, X is k-balanced. □

For example, let $k = m = 3$, and thus $X = [1, 18]$. Then $A_1 = \{1, 2, 3\}$, $A_2 = \{4, 5, 6\}$, and $A_3 = \{7, 8, 9\}$. $B_1 = \{16, 17, 18\}$, $B_2 = \{13, 14, 15\}$, and $B_3 = \{10, 11, 12\}$. The equipartition subsets of X are $C_1 = \{1, 2, 3, 16, 17, 18\}$, $C_2 = \{4, 5, 6, 13, 14, 15\}$, and $C_3 = \{7, 8, 9, 10, 11, 12\}$. Here, $\sum C_1 = \sum C_2 = \sum C_3 = 57$.

Corollary 2.1. *For any positive integers k, m, and p, the set $Y = [p + 1, 2km + p]$ is k-balanced.*

Proof. An equipartition of Y is $\{D_1, D_2, \ldots, D_k\}$, where $D_i = p+C_i$, $i \in [1, k]$, and C_i is defined as in the proof of Lemma 2.1. □

Theorem 2.1. *Let G be a graph with of order p and size q edges and admits a $K_{1,\Delta(G)}$-covering, where $\Delta(G)$ is maximum degree of G. Let H be a graph formed from G by attaching $m \geq 1$ pendants to every vertex v of G whose degree $deg(v) = \Delta(G)$. If G is $K_{1,\Delta(G)}$-supermagic, then H is $K_{1,\Delta(G)+m}$-supermagic.*

Proof. Let G be a $K_{1,\Delta(G)}$-supermagic graph with a $K_{1,\Delta(G)}$-supermagic labeling f. Let $v_1, v_2, \ldots,$ v_k be vertices of G such that $deg(v_i) = \Delta(G)$, $i \in [1, k]$. Then, for every $i \in [1, k]$ we have

$$c_f = f(v_i) + \sum_{u \in N(v_i)} f(u) + \sum_{u \in N(v_i)} f(uv_i),$$

where $N(v_i) = \{u : uv_i \in E(G)\}$.

Next, define H as a graph with

$$V(H) = V(G) \cup \{v_i^j : i \in [1, k], j \in [1, m]\},$$

$$E(H) = E(G) \cup \{v_i v_i^j : i \in [1, k], j \in [1, m]\}.$$

Thus, H is a graph of order $p+km$ and size $q+km$. Additionally, H is a graph with maximum degree $\Delta(G) + m$. Since G admits a $K_{1,\Delta(G)}$-covering and based on how H is constructed, then H admits a $K_{1,\Delta(G)+m}$-covering.

Let $U_1 = [1, p]$, $U_2 = [p + 1, 2km + p]$, and $U_3 = [2km + p + 1, 2km + p + q]$. So, U_1, U_2, U_3 is a partition of $[1, 2km + p + q]$. By corollary 1, the set $U_2 = [p + 1, 2km + p]$ is k-balanced. For every $i \in [1, k]$, let D_i be balanced subsets of U_2, where D_i is defined as in the proof of Lemma 2.

Next, define a total labeling

$$g : V(H) \cup E(H) \to [1, p + q + 2km]$$

as follows.

$$g(x) = \begin{cases} f(x), & \text{for } x \in V(G), \\ 2km + f(x), & \text{for } x \in E(G). \end{cases}$$

Under the labeling g, $g(V(G)) = [1, p]$ and $g(E(G)) = [2km + p + 1, 2km + p + q]$. Label the remaining $2km$ pendant vertices and $2km$ pendant edges of H, as follows. For $i \in [1, k]$, label $\{v_i^j : j \in [1, m]\} \cup \{v_i v_i^j : j \in [1, m]\}$ with the elements of D_i such that the label of v_i^j less than the label of $v_i v_i^j$. Thus, under the labeling g, $g(V(H)) = [1, p + km]$ and $g(E(H)) = [km + p + 1, p + q + 2km]$.

Next, we show that g is a $K_{1,\Delta(G)+m}$-supermagic labeling of H. For every $i \in [1, k]$,

$$\begin{aligned} c_g &= g(v_i) + \textstyle\sum_{u\in N(v_i)} g(u) + \sum_{u\in N(v_i)} g(uv_i) \\ &= g(v_i) + \textstyle\sum_{u\in N(v_i)\cap V(G)} g(u) + \sum_{u\in N(v_i)\cap V(G)} g(uv_i) \\ &\quad + \textstyle\sum_{j=1}^{m} g(v_i^j) + \sum_{j=1}^{m} g(v_i v_i^j) \\ &= f(v_i) + \textstyle\sum_{u\in N(v_i)} f(u) + \sum_{u\in N(v_i)} [2km + f(uv_i)] \\ &\quad + \textstyle\sum D_i \\ &= c_f + (2k\Delta(G) + 2p + 1)m + 2km^2. \end{aligned}$$

Hence, g is a $K_{1,\Delta(G)+m}$-supermagic labeling of H. So, H is a $K_{1,\Delta(G)+m}$-supermagic graph. □

Illustrations of Theorem 2.1 for case $\Delta(G) = 2, p = k = 5$, and $m = 1$ is given in Figure 1, and for case $\Delta(G) = 2, p = 7, k = 5$, and $m = 2$ is given in Figure 2.

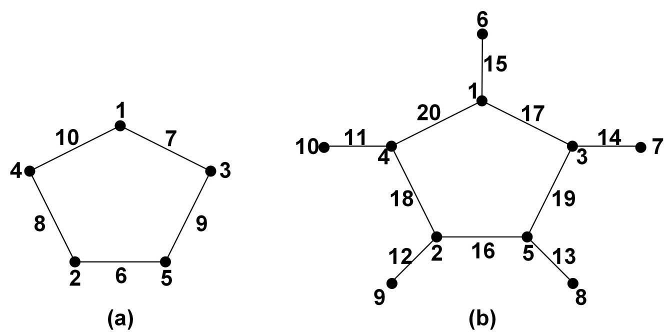

Figure 1. (a) The $K_{1,2}$-supermagic labeling of C_5 with the magic constant 25. (b) The $K_{1,3}$-supermagic labeling of the graph which is obtained by attaching a pendant to every vertex of C_5 with the magic constant 66.

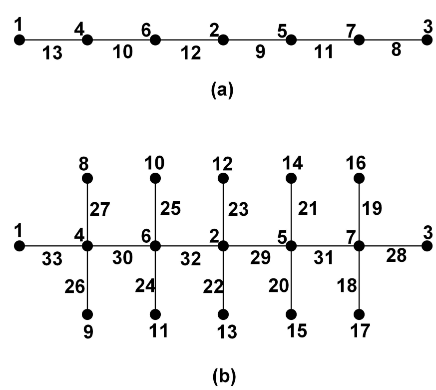

Figure 2. (a). A $K_{1,2}$-supermagic labeling of P_7 with the magic constant 34. (b) A $K_{1,4}$-supermagic labeling of a caterpillar which is formed by attaching two pendants to every vertices of P_7 except the pendants vertices with the magic constant 144.

In [3], Gutiérrez and Lladó proved the following results. The cycle C_n is P_t-supermagic for any $t \in [2, n-1]$ such that $gcd(n, t(t-1)) = 1$, and P_n is P_h-supermagic for every $h \in [2, n]$.

In particular, they proved that the cycle C_n is $P_3 \cong K_{1,2}$-supermagic for any $n > 3$ such that $gcd(n, 6) = 1$, and P_n is P_3-supermagic for every $n \geq 3$. As a consequence of these results and Theorem 2.1, we have the following corollaries.

Corollary 2.2. *For any $n > 3$ such that $gcd(n, 6) = 1$, and $m \geq 1$, the corona product of C_n and mK_1, $C_n \odot mK_1$, is a $K_{1,m+2}$-supermagic graph.*

Corollary 2.3. *For $n \geq 3$ and $m \geq 1$, the caterpillar formed by attaching m pendant edges to every vertex of degree two of the path P_n is a $K_{1,m+2}$-supermagic graph.*

The open problem related to the $K_{1,m+2}$-supermagic labeling of $C_n \odot mK_1$ is as follows.

Problem 1. *For any $n > 3$ such that $gcd(n, 6) \neq 1$, and $m \geq 1$, determine whether there is a $K_{1,m+2}$-supermagic labeling of $C_n \odot mK_1$.*

In [4], Jeyanthi and Selvagopal proved the following results.

Theorem 2.2. *[4] Let $H_1, H_2, \ldots, H_n$ be n disjoint copies of star $K_{1,n}$ and G_1 be the graph obtained by joining a new vertex to a pendant vertex of H_i, $i \in [1, n]$. Then G_1 is a $K_{1,n}$-supermagic graph.*

Theorem 2.3. *[4] Let $H_1, H_2, \ldots, H_{n+1}$ be $n+1$ disjoint copies of star $K_{1,n}$ and G_2 be the graph obtained by joining a new vertex to the center vertex of H_i, $i \in [1, n+1]$. Then G_2 is a $K_{1,n+1}$-supermagic graph.*

Again, as a consequence of these results and Theorem 2.1, we have the following corollaries.

Corollary 2.4. *For $m \geq 1$, the graphs G_1^* formed by attaching m pendant edges to every vertex of degree n of G_1 is a $K_{1,n+m}$-supermagic graph.*

Corollary 2.5. *For $m \geq 1$, the graphs G_2^* formed by attaching m pendant edges to every vertex of degree $n+1$ of G_2 is a $K_{1,n+m+1}$-supermagic graph.*

Next, we show the existence of a $K_{1,n}$-supermagic labeling of two classes of graphs for some integers n. Let $k \geq 1$ be an integer. Let G_k be a graph with $V(G_k) = \{x_i, y_i : i \in [1, k+2]\} \cup \{c_i : i \in [1, k+1]\}$ amd $E(G_k) = \{x_ic_i, y_ic_i : i \in [1, k+1]\} \cup \{c_ix_{i+1}, c_iy_{i+1} : i \in [1, k+1]\}$. Thus, G_k is a graph of order $3k+5$ and size $4k+4$, and it is obtained from a chain graph kC_4-path by attaching two pendants to the vertices c_1 and c_{k+1}, respectively.

Theorem 2.4. *For every positive integer $k \geq 1$, the graph G_k is $K_{1,4}$-supermagic.*

Proof. Define a vertex labeling $f_1 : V(G_k) \longrightarrow [1, 3k+5]$ as follows.

$$f_1(u) = \begin{cases} i, & \text{if } u = x_i,\ i \text{ is odd}, \\ i, & \text{if } u = y_i,\ i \text{ is even}, \\ \frac{1}{2}(3k+8-i), & \text{if } u = x_i,\ i \text{ is even},\ k \text{ is even}, \\ \frac{1}{2}(4k+9-i), & \text{if } u = y_i,\ i \text{ is odd},\ k \text{ is even}, \\ \frac{1}{2}(4k+10-i), & \text{if } u = x_i,\ i \text{ is even},\ k \text{ is odd}, \\ \frac{1}{2}(3k+8-i), & \text{if } u = y_i,\ i \text{ is odd},\ k \text{ is odd}, \\ 3k+6-i, & \text{if } u = c_i,\ i \in [1, k+1]. \end{cases}$$

Next, for every $i \in [1, k+1]$, define an edge labeling $f_2 : E(G_k) \longrightarrow [1, 4k+4]$ as follows.

$$f_2(u) = \begin{cases} 2i-1, & \text{if } u = x_i c_i, \\ 2i, & \text{if } u = c_i x_{i+1}, \\ 4k+6-2i, & \text{if } u = y_i c_i, \\ 4k+5-2i, & \text{if } u = c_i y_{i+1}. \end{cases}$$

For every $i \in [1, k+1]$, let $K_{1,4}^{(i)}$ be the sub-stars of G_k with vertex set $V(K_{1,4}^{(i)}) = \{c_i, x_i, x_{i+1}, y_i, y_{i+1}\}$ and edge set $E(K_{1,4}^{(i)}) = \{c_i x_i, c_i x_{i+1}, c_i y_i, c_i y_{i+1}\}$. It can be checked that for every $i \in [1, k+1]$,

$$f_1(K_{1,4}^{(i)}) = f_1(c_i) + f_1(x_i) + f_1(y_i) + f_1(x_{i+1}) + f_1(y_{i+1}) = \lfloor \frac{1}{2}(13k+31) \rfloor$$

and

$$f_2(K_{1,4}^{(i)}) = f_2(c_i x_i) + f_2(c_i y_i) + f_2(c_i x_{i+1}) + f_2(c_i y_{i+1}) = 8k+10.$$

Finally, define a total labeling $f_3 : V(G_k) \cup E(G_k) \longrightarrow [1, 7k+9]$ as follows.

$$f_3(u) = \begin{cases} f_1(u), & \text{if } u \in V(G_k), \\ 3k+5+f_2(u), & \text{if } u \in E(G_k). \end{cases}$$

It is easy to verify that, for every $i \in [1, k+1]$, $\sum f_3(K_{1,4}^{(i)}) = f_1(K_{1,4}^{(i)}) + 12k + 20 + f_2(K_{1,4}^{(i)}) = \lfloor \frac{1}{2}(53k+91) \rfloor$. □

As a direct consequence of this result and Theorem 2.1, we have the following corollary.

Corollary 2.6. *For any integers $k \geq 1$ and $m \geq 1$, the graph formed by attaching m pendants to every vertex of degree four of the graph G_k is a $K_{1,m+4}$-supermagic graph.*

Next, we consider of $K_{1,3}$-supermagic labelings of a ladder minus two edges. First, we define the ladder $L_n = P_n \times P_2$, $n \geq 3$, as a graph with vertex set $V(L_n) = \{x_i, y_i : i \in [1, n]\}$ and edge set $E(L_n) = \{x_i y_i : i \in [1, n]\} \cup \{x_i x_{i+1}, y_i y_{i+1} : i \in [1, n-1]\}$. For any integer $n \geq 3$, let $H_n = L_n - \{x_1 y_1, x_n y_n\}$. Thus, H_n is a graph with $V(H_n) = V(L_n)$ and $E(H_n) = E(L_n) - \{x_1 y_1, x_n y_n\}$. In the following theorem, we show that H_n is $K_{1,3}$-supermagic for every $n \geq 3$.

Theorem 2.5. *For any integer $n \geq 3$, H_n is $K_{1,3}$-supermagic.*

Proof. Define a vertex labeling $g_1 : V(H_n) \longrightarrow [1, 2n]$ as follows.
Case $n \equiv 0, 1 \bmod 4$.

$$g_1(u) = \begin{cases} \lfloor \frac{1}{2}(3n+3-i) \rfloor, & \text{if } u = x_i,\ i \equiv 0 \bmod 4, \\ \frac{1}{2}(i+1), & \text{if } u = x_i,\ i \equiv 1 \bmod 4, \\ \frac{1}{2}(4n+2-i), & \text{if } u = x_i,\ i \equiv 2 \bmod 4, \\ \lfloor \frac{1}{2}(n+2+i) \rfloor, & \text{if } u = x_i,\ i \equiv 3 \bmod 4, \\ \frac{1}{2}(4n+2-i), & \text{if } u = y_i,\ i \equiv 0 \bmod 4, \\ \lfloor \frac{1}{2}(n+2+i) \rfloor, & \text{if } u = y_i,\ i \equiv 1 \bmod 4, \\ \lfloor \frac{1}{2}(3n+3-i) \rfloor, & \text{if } u = y_i,\ i \equiv 2 \bmod 4, \\ \frac{1}{2}(i+1), & \text{if } u = y_i,\ i \equiv 3 \bmod 4. \end{cases}$$

Case $n \equiv 2, 3 \bmod 4$.

$$g_1(u) = \begin{cases} \frac{1}{2}(4n+2-i), & \text{if } u = x_i,\ i \equiv 0 \bmod 4, \\ \frac{1}{2}(i+1), & \text{if } u = x_i,\ i \equiv 1 \bmod 4, \\ \lfloor \frac{1}{2}(3n+3-i) \rfloor, & \text{if } u = x_i,\ i \equiv 2 \bmod 4, \\ \lfloor \frac{1}{2}(n+2+i) \rfloor, & \text{if } u = x_i,\ i \equiv 3 \bmod 4, \\ \lfloor \frac{1}{2}(3n+3-i) \rfloor, & \text{if } u = y_i,\ i \equiv 0 \bmod 4, \\ \lfloor \frac{1}{2}(n+2+i) \rfloor, & \text{if } u = y_i,\ i \equiv 1 \bmod 4, \\ \frac{1}{2}(4n+2-i), & \text{if } u = y_i,\ i \equiv 2 \bmod 4, \\ \frac{1}{2}(i+1), & \text{if } u = y_i,\ i \equiv 3 \bmod 4. \end{cases}$$

It is easy to verify that for $i \in [2, n-1]$, $g_1(x_{i-1}) + g_1(x_i) + g_1(x_{i+1}) + g_1(y_i) = g_1(y_{i-1}) + g_1(y_i) + g_1(y_{i+1}) + g_1(x_i)$ is $4n+3$, if n is even and $4n+4$, if n is odd.

Next, define an edge labeling $g_2 : E(H_n) \longrightarrow [1, 3n-4]$ as follows.

$$g_2(u) = \begin{cases} \frac{1}{2}(i+1), & \text{if } u = x_i x_{i+1},\ i \text{ is odd}, \\ \lfloor \frac{1}{2}(3n-2+i) \rfloor, & \text{if } u = x_i x_{i+1},\ i \text{ is even}, \\ \frac{1}{2}(2n-1+i), & \text{if } u = y_i y_{i+1},\ i \text{ is odd}, \\ \lfloor \frac{1}{2}(n+i) \rfloor, & \text{if } u = y_i y_{i+1},\ i \text{ is even}, \\ 3n-2-i, & \text{if } u = x_i y_i,\ i \in [2, n-1]. \end{cases}$$

It can be checked that for $i \in [2, n-1]$, $g_1(x_{i-1}x_i) + g_1(x_i x_{i+1}) + g_1(x_i y_i) = g_1(y_{i-1}y_i) + g_1(y_i y_{i+1}) + g_1(y_i x_i)$ is $\frac{1}{2}(9n-6)$, if n is even and $\frac{1}{2}(9n-7)$, if n is odd.

At last, define a total labeling $g_3 : V(H_n) \cup E(H_n) \longrightarrow [1, 5n-4]$ as follows.

$$g_3(u) = \begin{cases} g_1(u), & \text{if } u \in V(H_n), \\ 2n + g_2(u), & \text{if } u \in E(H_n). \end{cases}$$

It is a routine procedure to check that g_3 is a $K_{1,3}$-supermagic labeling of H_n where for every subgraph H' of H_n isomorphic to $K_{1,3}$, $\sum f_3(H')$ is $\lfloor \frac{1}{2}(29n+1) \rfloor$. □

By applying Theorem 2.1 to this result, we have the following result.

Corollary 2.7. *For any integers $n \geq 3$ and $m \geq 1$, the graph formed by attaching m pendant edges to every vertex of degree three of the graph H_n is a $K_{1,m+3}$-supermagic graph.*

Problem 2. *Investigate the existence of $K_{1,n}$-supermagic labelings of other classes of graphs.*

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